Power Coupling Equations for Single-Mode, Single-Polarization Optical Fibers with Effects of Leaky Modes and Broad-Band Light Source

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***Abstract*— A set of coupled equations for single-mode single-polarization (SMSP) optical fibers applicable to the case with effects of leaky modes and broad-band light source has been derived. The power coupling coefficients are related not only to the birefringence of the SMSP fiber, but also to the leaky mode loss and the spectral line width of the light source.**

THE authors have analyzed the polarization mode coupling in an SMSP fiber when a monochromatic light propagates in the fiber [1]. The analysis was based on the fact that leaky modes exist in an SMSP fiber with finite cladding, i.e., it considered the influence of the leaky outer jacket [2]. This letter presents a further theoretical analysis of the polarization coupling in an SMSP fiber in which a broad-band (but quasi-monochromatic) light is propagated. A random distribution of the external perturbation along the fiber is assumed, which causes the random polarization coupling. This distribution is characterized by an exponential-function shaped autocorrelation function. In addition, the spectral density of a Lorentzian-shape is assumed to characterize the quasi-monochromatic light source.

Two fundamental polarization modes exist in an SMSP fiber: a guided mode with relatively low loss and a leaky mode which is strongly attenuated. When the SMSP fiber undergoes external perturbations, the polarization coupling is induced, resulting in a continuous exchange of energy between the guided and leaky modes. A part of the guided mode energy is coupled to the leaky mode, and vice versa, whereas, in addition, a part of the leaky mode energy leaks to the cladding to be attenuated.

The coupled amplitude equations for a monochromatic light source have been given in [1]. From these equations and using Marcuse's methods, also considering the effect of broad-band light source, we obtain the coupled power equations with broad-band light source as follows:

$$\frac{dP_1}{dz} = -2\alpha P_1 + h_{11}P_1 + h_{12}P_2,$$
$$\frac{dP_2}{dz} = h_{21}P_1 + h_{22}P_2,$$

where

$$h_{11} = \hat{K}_{12}\hat{K}_{21}\int_0^\infty R(u)T(Du) \cdot \exp[(j\beta_y - j\beta_x + \alpha)u]du + \text{ c.c.,} \tag{1a}$$

$$h_{12} = \left|\hat{K}_{12}\right|^2\int_0^\infty R(u)T(Du) \cdot \exp[(j\beta_y - j\beta_x - \alpha)u]du + \text{ c.c.,} \tag{1b}$$

$$h_{21} = \left|\hat{K}_{21}\right|^2\int_0^\infty R(u)T(Du) \cdot \exp[(j\beta_x - j\beta_y + \alpha)u]du + \text{ c.c.,} \tag{2a}$$

$$h_{22} = \hat{K}_{12}\hat{K}_{21}\int_0^\infty R(u)T(Du) \cdot \exp[(j\beta_x - j\beta_y - \alpha)u]du + \text{ c.c.,} \tag{2b}$$

The symbol c.c. indicates that the complex conjugate of the preceding term should be added. P_1 is the statistical average power carried by the fundamental leaky mode and P_2 is the fundamental guided mode. $R(u)$ is an autocorrelation function of the disturbance function of the envelop function of incident electric field. We assume that the random distribution of perturbation is a stationary process characterized by an exponential-function shaped autocorrelation function:

$$R(u) = \sigma_0^2 \exp(-|u|/L), \tag{3}$$

where σ_0 is the variance of the disturbance and L is the correlation length. The autocorrelation function $T(s)$ of the

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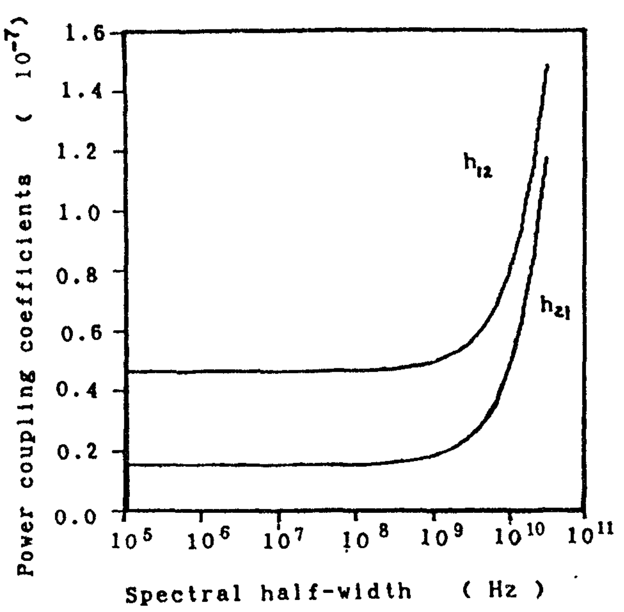

Fig. 1. Power coupling coefficients as a function of the spectral half-width of a Lorentzian spectrum.

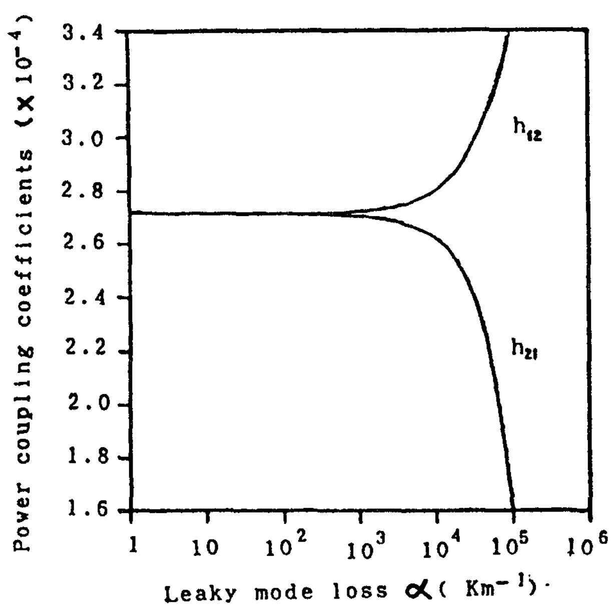

Fig. 2. Power coupling coefficients as a function of the leaky mode lossα for a Lorentzian spectrum, when $\Delta\gamma = 10^{14}$ Hz, $L = 10$ m, and $B = 1 \times 10^{-4}$.

light is determined by the spectral density $G(\omega)$ of the light. We assume here that $G(\omega)$ is expressed in a Lorentzian form:

$$G(\omega) = (\Delta\omega)^2 / \left[(\omega - \omega_0)^2 + (\Delta\omega)^2\right],$$

as has been demonstrated with laser diodes [3]. The autocorrelation function $T(s)$ of the light is then expressed as

$$T(s) = \exp(-|\Delta\omega| s), \tag{4}$$

where $2\Delta\gamma = \Delta\omega/\pi$ denotes the spectral width of the light source.

From this analysis, we can calculate the power coupling coefficients of an SMSP fiber. Combining (1b), (2a), (3), and (4), we obtain the power coupling coefficients for the Lorentzian spectrum as

$$h_{12} = \left|\hat{K}_{12}\right|^2 \sigma_0^2 \left(\frac{1}{L} + D|\Delta\omega| + \alpha\right) / \left[(\beta_y - \beta_x)^2 + \left(\frac{1}{L} + D|\Delta\omega| + \alpha\right)^2\right],$$

$$h_{21} = \left|\hat{K}_{21}\right|^2 \sigma_0^2 \left(\frac{1}{L} + D|\Delta\omega| - \alpha\right) / \left[(\beta_y - \beta_x)^2 + \left(\frac{1}{L} + D|\Delta\omega| - \alpha\right)^2\right].$$

Fig. 1 shows the coupling coefficients in h_{12} and h_{21} as a function of the spectral half-width of light $\Delta\gamma(2\Delta\gamma = \Delta\omega/\pi)$, when the light source has a Lorentzian spectrum. In this figure, it is assumed that correlation length $L = 50$ m, $|\hat{K}_{12}|^2\sigma_0^2 = |\hat{K}_{21}|^2\sigma_0^2 = 0.36$, the birefringence $B = 1 \times 10^{-4}(\beta_y - \beta_x = 2\pi B/\lambda)$, the leaky mode loss $\alpha = 10$ (km^{-1}), the polarization mode dispersion value $D = B/C$, and the wavelength $\lambda = 1.3$ μm. Fig. 1 demonstrates that both of the power coupling coefficients h_{12} and h_{21} increase as $\Delta\gamma$ increases, but h_{12} increases faster than h_{21}. The difference between h_{12} and h_{21} also becomes smaller as $\Delta\gamma$ increases. Therefore, the larger the spectral half-width of the light source, the weaker the single polarization property of the SMSP fibers.

Fig. 2 shows the computed coupling coefficients as a function of the leaky mode loss α. The two curves indicate that h_{12} increases but h_{21} decreases as α increases. This means that more energy flows from the guided mode to the leaky mode than in the opposite direction when α is increased. Thus, the guided mode will experience an additional loss due to the polarization mode coupling.

We have derived the coupled power equations for an SMSP optical fiber assuming a broad-band light source. In general, the power coupling coefficient h_{12} is not equal to h_{21}, in other words, the polarization coupling is not a reciprocal process, i.e., the energy is exchanged unequally between the two polarization modes. The coupling coefficient h_{12} is always greater than h_{21}, which means that the guided mode couples more to the leaky mode than vice versa. For a light source having a Lorentzian-shaped spectrum, the single polarization property of an SMSP fiber will become weakened when spectral half-width $\Delta\gamma$ increases.

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